

第6回 小テスト (積分)

学生番号 _____

氏名 _____

1. 次の不定積分を求めなさい.

$$(1) \int 2x \sin x dx$$

答.

$$\begin{aligned} \int 2x \sin x dx &= \int 2x(-\cos x)' dx \\ &= 2x(-\cos x) - \int (2x)'(-\cos x) dx \\ &= -2x \cos x - \int 2 \cdot (-\cos x) dx \\ &= -2x \cos x + 2 \sin x + C \quad (C \text{ は積分定数}) \end{aligned}$$

$$(2) \int x e^{2x} dx$$

答.

$$\begin{aligned} \int x e^{2x} dx &= \int x \left(\frac{1}{2} e^{2x} \right)' dx \\ &= x \cdot \frac{1}{2} e^{2x} - \int (x)' \cdot \frac{1}{2} e^{2x} dx \\ &= \frac{1}{2} x e^{2x} - \frac{1}{2} \int e^{2x} dx \\ &= \frac{1}{2} x e^{2x} - \frac{1}{4} e^{2x} + C \\ &= \left(\frac{1}{2} x - \frac{1}{4} \right) e^{2x} + C \quad (C \text{ は積分定数}) \end{aligned}$$

$$(3) \int e^{2x} \sin x dx$$

答. $I = \int e^{2x} \sin x dx$ と置くと,

$$\begin{aligned} I &= \int e^{2x} \sin x dx = \int e^{2x} (-\cos x)' dx \\ &= e^{2x} \cdot (-\cos x) - \int (e^{2x})' \cdot (-\cos x) dx \\ &= -e^{2x} \cos x - \int 2e^{2x} \cdot (-\cos x) dx \\ &= -e^{2x} \cos x - \int 2e^{2x} \cdot (-\sin x)' dx \\ &= -e^{2x} \cos x - \left(2e^{2x} \cdot (-\sin x) - \int (2e^{2x})' \cdot (-\sin x) dx \right) \\ &= -e^{2x} \cos x + 2e^{2x} \sin x - 4 \int e^{2x} \sin x dx \\ &= (-\cos x + 2 \sin x) e^{2x} - 4I \end{aligned}$$

従って, $5I = (-\cos x + 2 \sin x) e^{2x}$ である. よって,

$$I = \frac{1}{5} (-\cos x + 2 \sin x) e^{2x}$$

$$(4) \int \log(x+2) dx$$

答.

$$\begin{aligned} \int \log(x+2) dx &= \int \log(x+2) \cdot (x+2)' dx \\ &= \log(x+2) \cdot (x+2) - \int (\log(x+2))' \cdot (x+2) dx \\ &= (x+2) \log(x+2) - \int \frac{1}{x+2} \cdot (x+2) dx \\ &= (x+2) \log(x+2) - \int dx \\ &= (x+2) \log(x+2) - x + C \quad (C \text{ は積分定数}) \end{aligned}$$